Part I

Answer all 24 questions in this part. Each correct answer will receive 2 credits. No partial credit will be allowed. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For each statement or question, choose the word or expression that, of those given, best completes the statement or answers the question. Record your answers on your separate answer sheet. [48]

Use this space for computations.

1 Which equation represents the line that passes through the point $(-2,2)$ and is parallel to $y = \frac{1}{2}x + 8$?

(1) $y = \frac{1}{2}x$

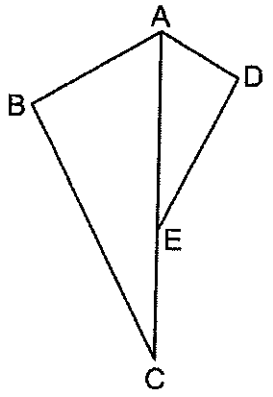
(3) $y = \frac{1}{2}x + 3$

(2) $y = -2x - 3$

(4) $y = -2x + 3$

$y - y = m(x - x)$
$y - 2 = \frac{1}{2}(x - -2)$
$y - 2 = \frac{1}{2}(x + 2)$
$y - 2 = \frac{1}{2}x + 1$
$+2 \quad +2$
$y = \frac{1}{2}x + 3$

2 In the diagram below, $\triangle ADE$ is the image of $\triangle ABC$ after a reflection over the line AC followed by a dilation of scale factor $\frac{AE}{AC}$ centered at point A.



$A \rightarrow A$
$B \rightarrow D$
$C \rightarrow E$

Which statement must be true?

(1) $m\angle BAC = m\angle AED$

(3) $m\angle DAE = \frac{1}{2}m\angle BAC$

$m\angle ABC = m\angle ADE$

(4) $m\angle ACB = \frac{1}{2}m\angle DAB$

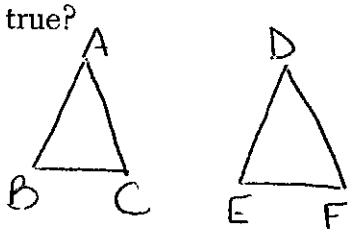
3 Given $\triangle ABC \cong \triangle DEF$, which statement is *not* always true?

$\overline{BC} \cong \overline{DF}$

(2) $m\angle A = m\angle D$

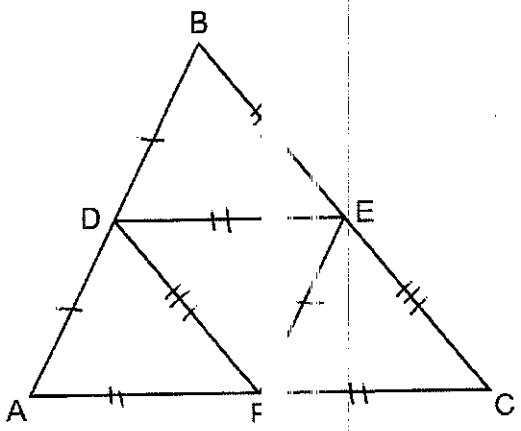
(3) area of $\triangle ABC$ = area of $\triangle DEF$

(4) perimeter of $\triangle ABC$ = perimeter of $\triangle DEF$



Use this space for computations.

4 In the diagram below, $\overline{DE}$, $\overline{DF}$, a $\overline{EF}$ are midsegments of $\triangle ABC$.



The perimeter of quadrilateral A EF is equivalent to

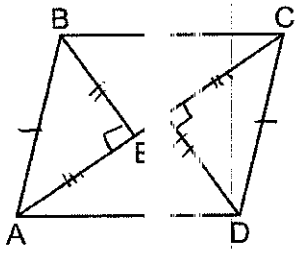
(1) $AB + BC + AC$

(2) $\frac{1}{2}AB + \frac{1}{2}AC$

($2AB + 2AC$

($AB + AC$

5 In the diagram below, if $\triangle ABE$ $\triangle CDF$ and $\overline{AEFC}$ is drawn, then it could be proven that quadrilat al $ABCD$ is a



(1) square

(2) rhombus

(rectangle

(parallelogram

6 Under which transformation wou l $\triangle A'B'C'$, the image of $\triangle ABC$, *not* be congruent to $\triangle ABC$?

(1) reflection over the y-axis

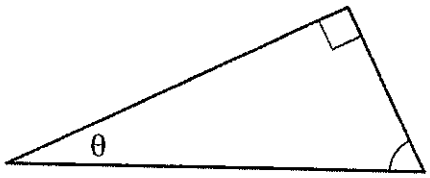
(2) rotation of 90° clockwise ab t the origin

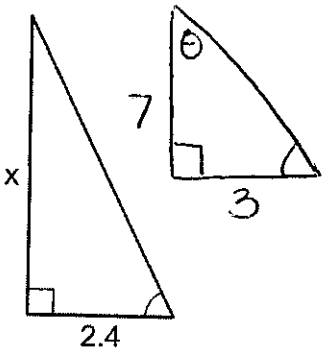
(3) translation of 3 units right a l 2 units down

(dilation with a scale factor o 2 centered at the origin

7 The diagram below shows two similar triangles.

Use this space for computations.





$\frac{x}{2.4} = \frac{7}{3}$

$\frac{3x}{3} = \frac{16.8}{3}$

$x = 5.6$

If $\tan\theta = \frac{3}{7}$ ($\frac{opp}{adj}$), what is the value of x, to the *nearest tenth*?

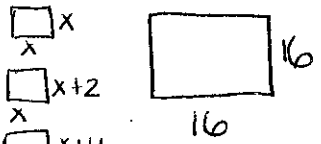
(1) 1.2
(2) 5.6
(3) 7.6
(4) 8.8

8 A farmer has 64 feet of fence to enclose a rectangular vegetable garden. Which dimensions would result in the biggest area for this garden?

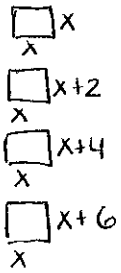
(1) the length and the width are equal
(2) the length is 2 more than the width
(3) the length is 4 more than the width
(4) the length is 6 more than the width

$\frac{64}{4} = 16$

choice(1)



$16(16) = 256$



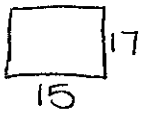
choice(2)

$2x + 2(x+2) = 64$

$2x + 2x + 4 = 64$

$4x = 60$

$x = 15$



$15(17) = 255$

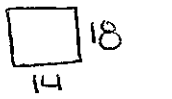
choice (3)

$2x + 2(x+4) = 64$

$2x + 2x + 8 = 64$

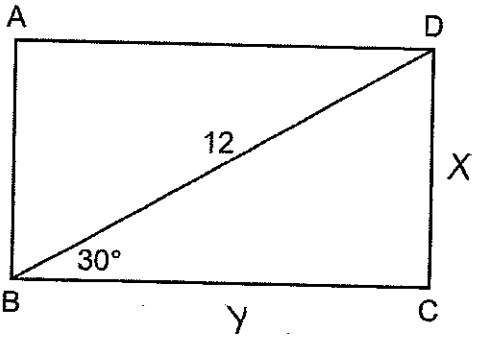
$4x = 56$

$x = 14$



$14(18) = 252$

9 The diagram shows rectangle $ABCD$, with diagonal $\overline{BD}$.



$\sin 30 = \frac{x}{12}$

$x = 6$

$\cos 30 = \frac{y}{12}$

$y = 10.3923$

$P = 2(6) + 2(10.3923)$

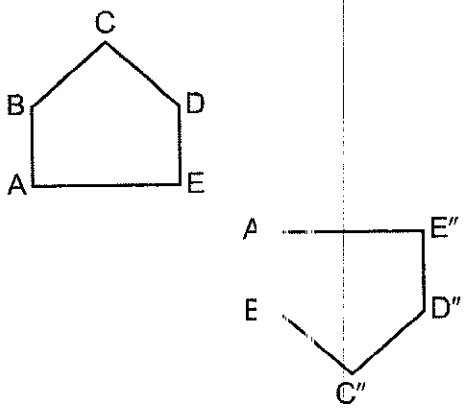
$P = 32.8$

What is the perimeter of rectangle $ABCD$, to the *nearest tenth*?

(1) 28.4
(2) 32.8
(3) 48.0
(4) 62.4

Use this space for computations.

10 Identify which sequence of transformations could map pentagon $ABCDE$ onto pentagon $A''B''C''D''E''$, as shown below.



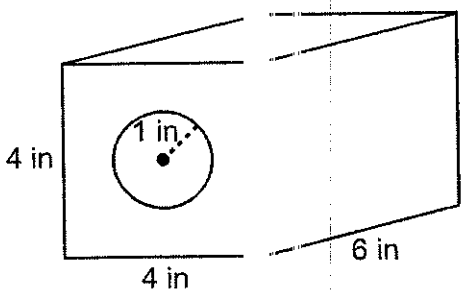
(1) dilation followed by a rotation

(2) translation followed by a rotation

(3) line reflection followed by a translation

(4) line reflection followed by a line reflection

11 A solid metal prism has a rectangular base with sides of 4 inches and 6 inches, and a height of 4 inches. A hole in the shape of a cylinder, with a radius of 1 inch, is drilled through the entire length of the rectangular prism.



$V_{prism} - V_{cylinder}$

$lwh - \pi r^2 h$

$4(6)(4) - \pi(1)^2(6)$

$96 - 18.84955592$

77

What is the approximate volume of the remaining solid, in cubic inches?

(1) 19

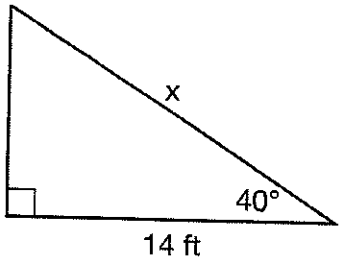
(2) 93

(3) 77

(4) 96

12 Given the right triangle in the diagram below, what is the value of x, to the *nearest foot*?

Use this space for computations.



$\frac{\cos 40}{1} = \frac{14}{x}$

$x \cdot \frac{\cos 40}{\cos 40} = \frac{14}{\cos 40}$

$x = 18$

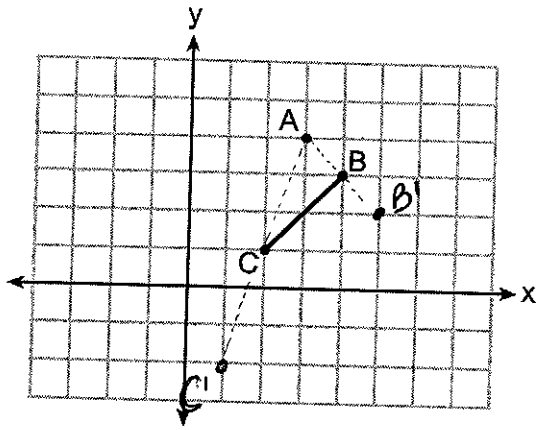
(1) 11

(2) 17

(3) 18

(4) 22

13 On the graph below, point $A(3,4)$ and $\overline{BC}$ with coordinates $B(4,3)$ and $C(2,1)$ are graphed.



What are the coordinates of B' and C' after $\overline{BC}$ undergoes a dilation centered at point A with a scale factor of 2?

(1) $B'(5,2)$ and $C'(1,-2)$

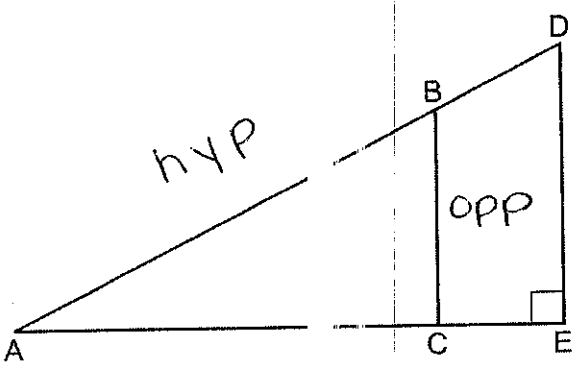
(2) $B'(6,1)$ and $C'(0,-1)$

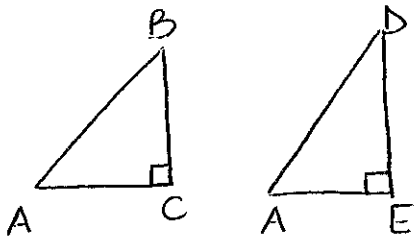
(3) $B'(5,0)$ and $C'(1,-2)$

(4) $B'(5,2)$ and $C'(3,0)$

Use this space for computations.

14 In the diagram of right triangle ADE below, $\overline{BC} \parallel \overline{DE}$.





$\sin A = \frac{BC}{AB}$

Which ratio is always equivalent to the sine of $\angle A$?

(1) $\frac{AD}{DE}$

(3) $\frac{BC}{AB}$

(2) $\frac{AE}{AD}$

(4) $\frac{AB}{AC}$

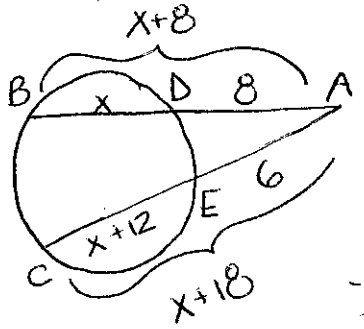
15 In circle O, secants $\overline{ADB}$ and $\overline{AE}$ are drawn from external point A such that points D, B, E, and C are on circle O. If $AD = 8$, $AE = 6$, and EC is 12 more than BD, the length of $\overline{BD}$ is

(1) 6

(3) 36

(2) 22

(4) 48



$8(X+8) = 6(X+18)$

$8X+64 = 6X+108$

$-6X \quad -6X$

$2X+64 = 108$

$-64 \quad -64$

$\frac{2X}{2} = \frac{44}{2}$

$X = 22$

16 A parallelogram is always a rectangle if

(2) the diagonals are congruent

(2) the diagonals bisect each other

(3) the diagonals intersect at right angles

(4) the opposite angles are congruent

Use this space for computations.

17 Which rotation about its center will carry a regular decagon onto itself?

(1) 54°
(2) 162°
(3) 198°
● 252°

$\frac{360}{10} = 36, 72, 108 \ldots 252$

18 The equation of a circle is $x^2 + y^2 - 6y + 1 = 0$. What are the coordinates of the center and the length of the radius of this circle?

$\left(\frac{-6}{2}\right)^2 = (-3)^2 = 9$

● center (0,3) and radius = $2\sqrt{2}$
(2) center (0,−3) and radius = $2\sqrt{2}$
(3) center (0,6) and radius = $\sqrt{35}$
(4) center (0,−6) and radius = $\sqrt{35}$

$x^2 + y^2 - 6y + 9 = -1 + 9$

$x^2 + (y-3)^2 = 8$

$C = (0,3)$ $\sqrt{r^2} = \sqrt{8}$

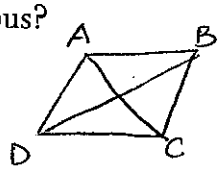
$r = \sqrt{8}$

$\sqrt{4}\sqrt{2}$

$2\sqrt{2}$

19 Parallelogram $ABCD$ has coordinates $A(0,7)$ and $C(2,1)$. Which statement would prove that $ABCD$ is a rhombus?

(1) The midpoint of $\overline{AC}$ is (1,4).
(2) The length of $\overline{BD}$ is $\sqrt{40}$.
● The slope of $\overline{BD}$ is $\frac{1}{3}$.
(4) The slope of $\overline{AB}$ is $\frac{1}{3}$.



Diagonals are ⊥

$AC = \frac{7-1}{0-2} = \frac{6}{-2} = -3$

$-3 \perp \frac{1}{3}$

20 Point Q is on $\overline{MN}$ such that $MQ:QN = 2:3$. If M has coordinates (3,5) and N has coordinates (8,−5), the coordinates of Q are

● (5,1)
(2) (5,0)
(3) (6,−1)
(4) (6,0)

$M(3,5) \xrightarrow{T\langle 5,-10\rangle} N(8,-5)$

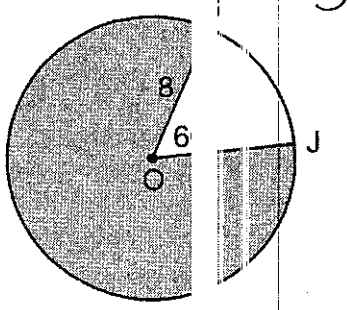
$5\left(\frac{2}{5}\right) \quad -10\left(\frac{2}{5}\right)$

$M(3,5) \xrightarrow{T\langle 2,-4\rangle} Q(5,1)$

Use this space for computations.

21 In the diagram below of circle O, $O = 8$ and $m\angle GOJ = 60°$.

360 - 60 = 300



$A.S. = \frac{n}{360} \cdot \pi r^2$

$\frac{300}{360} \cdot \pi(8)^2 = \frac{160\pi}{3}$

What is the area, in terms of π, of the shaded region?

(1) $\frac{4\pi}{3}$

(2) $\frac{20\pi}{3}$

(3) $\frac{32\pi}{3}$

(4) $\frac{160\pi}{3}$

22 A circle whose center is the origin passes through the point $(-5,12)$. Which point also lies on this circle?

$(x-h)^2+(y-k)^2=r^2$ $\quad x^2+y^2=169$

$(-5-0)^2+(12-0)^2=r^2$

$25+144=r^2$

$\sqrt{169}=\sqrt{r^2}$

$13=r$

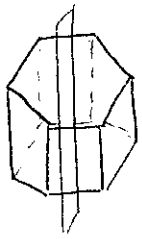
(1) $(10,3)$
$(10)^2+(3)^2=109$

(2) $(-12,13)$
$(-12)^2+(13)^2=313$

(3) $(1.1, 2\sqrt{12})$
$(1)^2+(2\sqrt{12})^2=169$

(4) $(-8, 5\sqrt{21})$
$(-8)^2+(5\sqrt{21})^2=589$

23 A plane intersects a hexagonal prism. The plane is perpendicular to the base of the prism. Which two-dimensional figure is the cross section of the plane intersecting the prism?



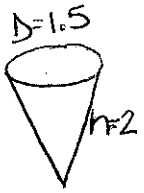
(1) triangle

(2) trapezoid

(3) hexagon

(4) rectangle

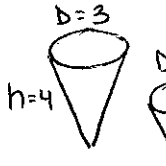
24 A water cup in the shape of a cone has a height of 4 inches and a maximum diameter of 3 inches. What is the volume of the water in the cup, to the *nearest tenth of a cubic inch*, when the cup is filled to half its height?



$V=\frac{1}{3}\pi r^2 h$

$V=\frac{1}{3}\pi(.75)^2(2)$

$V=1.2$



$\frac{3}{4}=\frac{x}{2}$

$4x=6$

$x=1.5$

(1) 1.2

(2) 3.5

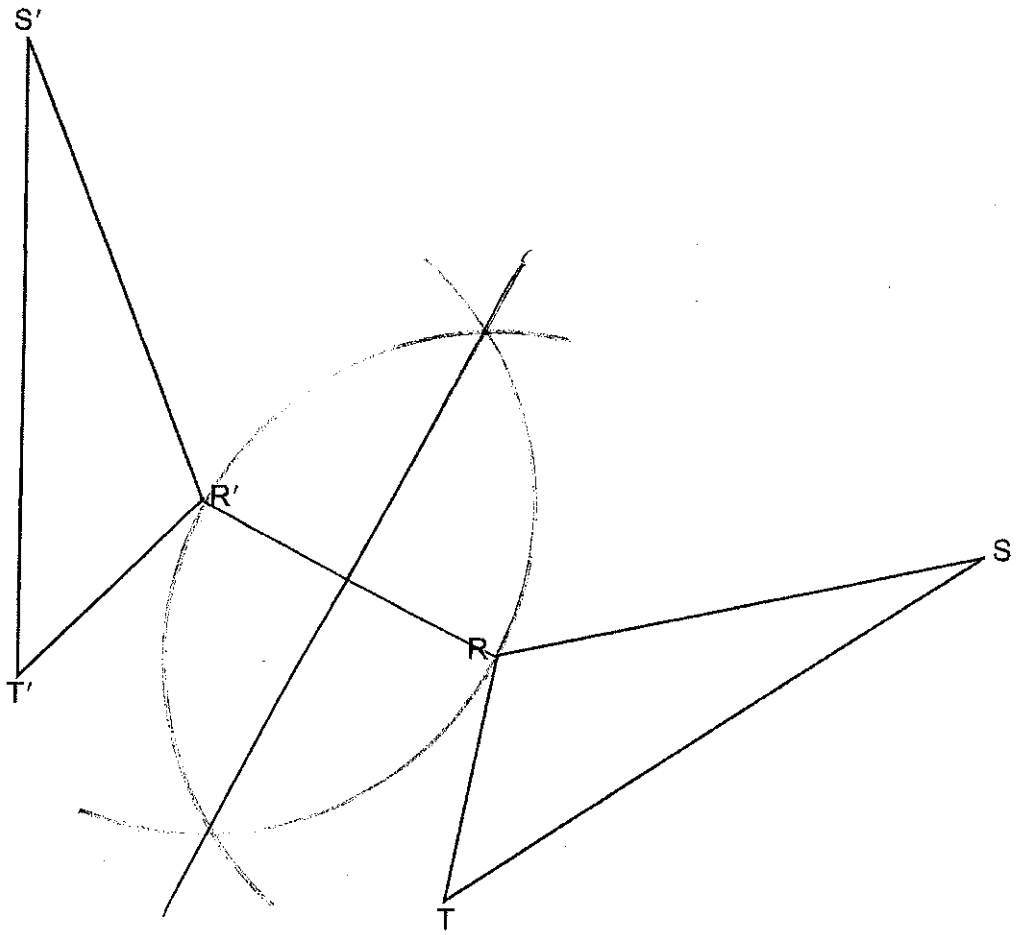
(3) 4.7

(4) 14.1

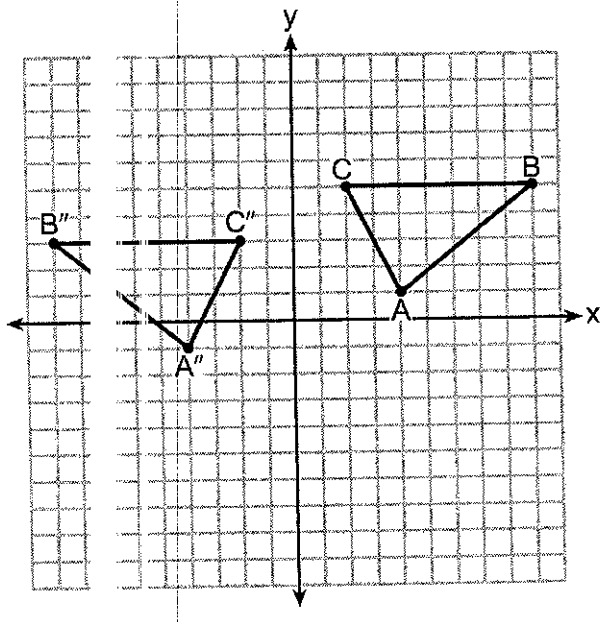
Part II

Answer all 7 questions in this part. Each correct answer will receive 2 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [14]

25 Using a compass and straightedge, construct the line of reflection over which triangle RST reflects onto triangle $R'S'T'$. [Leave all construction marks.]



26 The graph below shows $\triangle ABC$ and its image, $\triangle A''B''C''$.



Describe a sequence of rigid motions which would map $\triangle ABC$ onto $\triangle A''B''C''$.

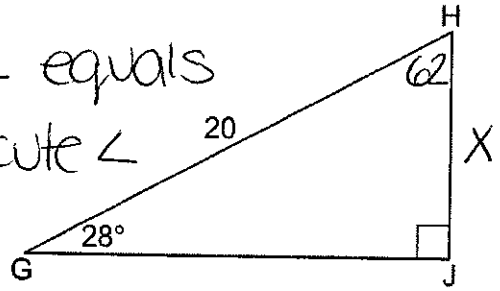
reflection over the y-axis

translation down 2

27 When instructed to find the length of $\overline{HJ}$ in right triangle HJG, Alex wrote the equation $\sin 28° = \frac{HJ}{20}$ while Marlene wrote $\cos 62° = \frac{HJ}{20}$. Are both students' equations correct? Explain why.

180-90-28=62

Sin of one acute ∠ equals
Cos of the other acute ∠

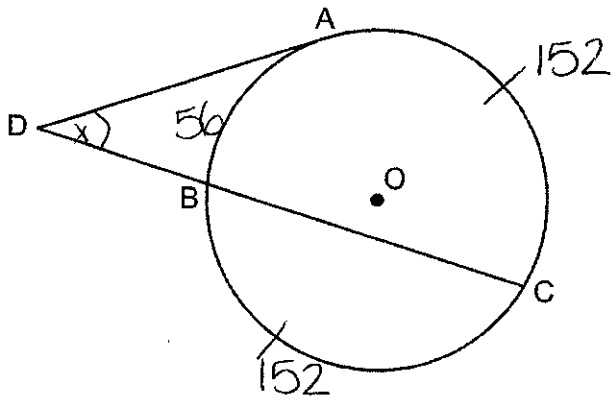


$\sin 28 = \frac{X}{20}$ $\quad$ $\cos 62 = \frac{X}{20}$

$X = 9.389431$ $\quad$ $X = 9.389431$

28 In the diagram below, tangent $\overline{DA}$ and secant $\overline{DBC}$ are drawn to circle O from external point D, such that $\overset{\frown}{AC} \cong \overset{\frown}{BC}$.

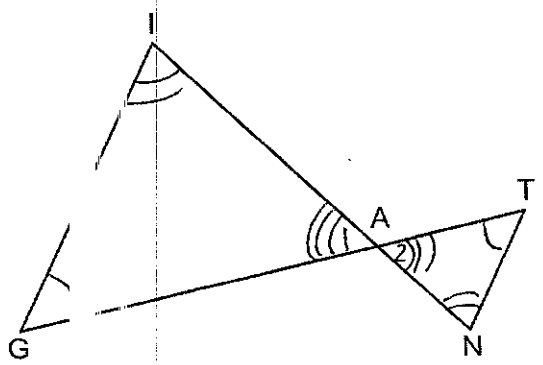
360
-304
56



If $m\overset{\frown}{BC} = 152°$, determine and state $m\angle D$.

$X = \frac{152-56}{2} = \frac{96}{2} = \boxed{48}$

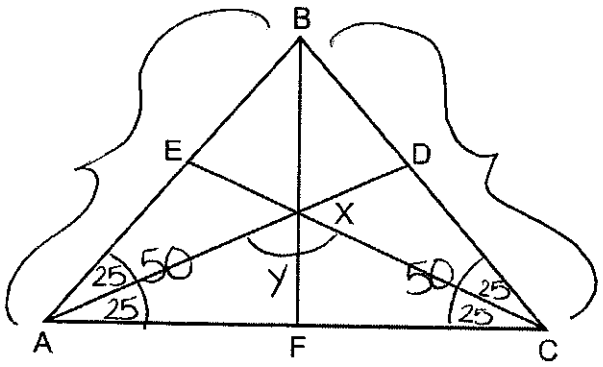
29 In the diagram below, $\overline{GI}$ is parallel to $\overline{NT}$, and $\overline{IN}$ intersects $\overline{GT}$ at A.



Prove: $\triangle GIA \sim \triangle TNA$

S	R
① $\overline{GI} \parallel \overline{NT}$, $\overline{IN}$ intersects $\overline{GT}$	① Given
② $\angle G \cong \angle T$, $\angle I \cong \angle N$	② || lines cut by a transversal form $\cong$ alternate interior $\angle$s
③ $\angle 1 \cong \angle 2$	③ Intersecting lines form $\cong$ vertical $\angle$s
④ $\triangle GIA \sim \triangle TNA$	④ AA $\sim$

30 In the diagram below of isosceles triangle ABC, $\overline{AB} \cong \overline{CB}$ and angle bisectors $\overline{AD}$, $\overline{BF}$, and $\overline{CE}$ are drawn and intersect at X.



If $m\angle BAC = 50°$, find $m\angle AXC$.

$25 + 25 + y = 180$

$50 + y = 180$

$\boxed{y = 130}$

31 In square *GEOM*, the coordinat of *G* are (2,−2) and the coordinates of *O* are (−4,2). Determine and state the coordi ates of vertices *E* and *M*.

[The use of the set of axes belo is optional.]

1

$GO = \frac{4}{-6} = \frac{-2}{3}$

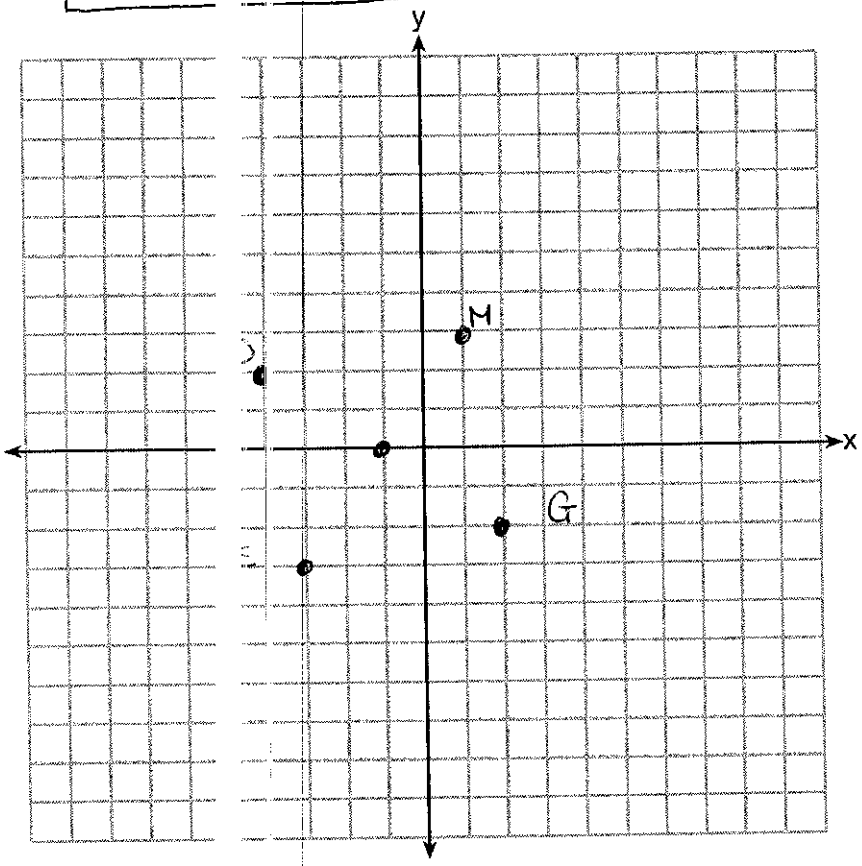
$EM = \frac{3}{2}$

Midpt GO $\left(\frac{2+-4}{2}, \frac{-2+2}{2}\right)$

$\left(\frac{-2}{2}, \frac{0}{2}\right)$

$(-1,0)$

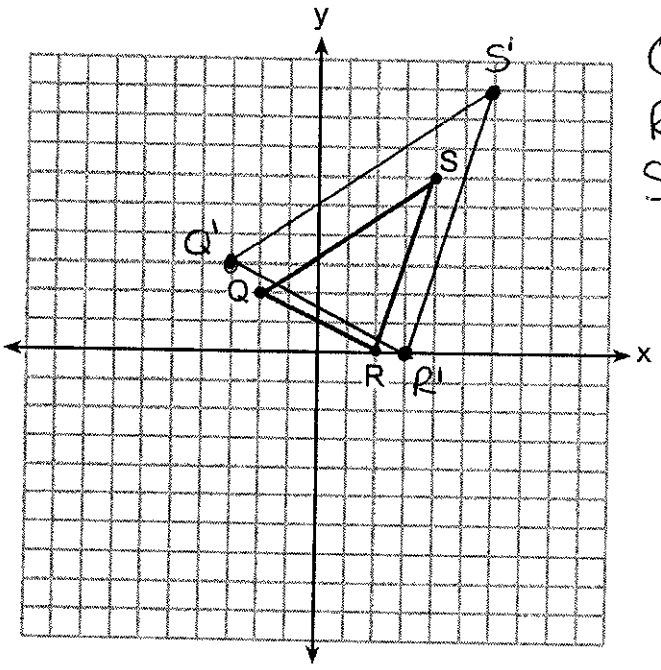
E(-3, 3) M(1,3)



Part III

Answer all 3 questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the information provided for each question to determine your answer. Note that diagrams are not necessarily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs and drawings, which should be done in pencil. [12]

32 Triangle QRS is graphed on the set of axes below.



$Q(-2,2) \rightarrow Q'(-3,3)$
$R(2,0) \rightarrow R'(3,0)$
$S(4,6) \rightarrow S'(6,9)$

On the same set of axes, graph and label $\triangle Q'R'S'$, the image of $\triangle QRS$ after a dilation with a scale factor of $\frac{3}{2}$ centered at the origin.

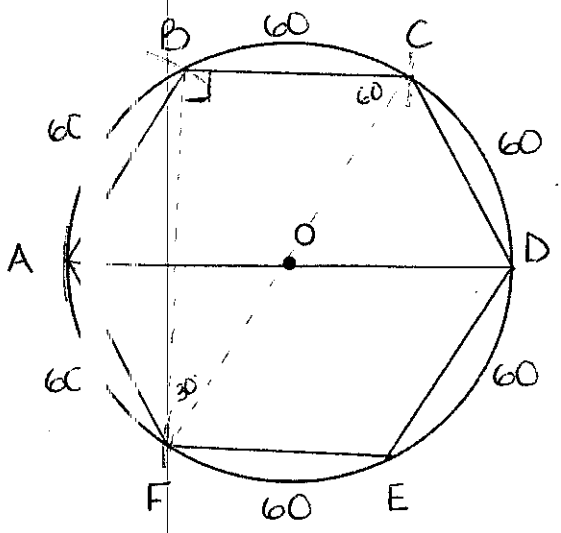
Use slopes to explain why $\overline{Q'R'} \parallel \overline{QR}$.

$QR = \frac{2-0}{-2-2} = \frac{2}{-4} = -\frac{1}{2}$

$Q'R' = \frac{3-0}{-3-3} = \frac{3}{-6} = -\frac{1}{2}$

Same slope

33 Using a compass and straighte ge, construct a regular hexagon inscribed in circle O below. Label it $ABCDEF$. [Leave all c struction marks.]



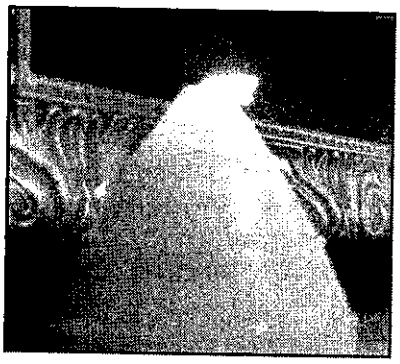
$\frac{360}{6} = 60$

If chords $\overline{FB}$ and $\overline{FC}$ are draw , which type of triangle, according to its angles, would $\triangle FBC$ be? Explain your answer.

Right Δ

∠ inscribed in a semi-circle is a right ∠

34 A candle maker uses a mold to make candles like the one shown below.



The height of the candle is 13 cm and the circumference of the candle at its widest measure is 31.416 cm. Use modeling to approximate how much wax, to the *nearest cubic centimeter,* is needed to make this candle. Justify your answer.

$C = \frac{\pi d}{\pi} = \frac{31.416}{\pi} \approx 10.000$ $\quad r = 5$

$V_{cone} = \frac{1}{3}\pi r^2 h$

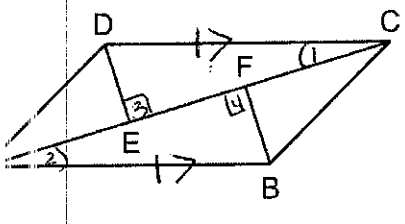
$\frac{1}{3}\pi(5)^2(13)$

$\boxed{340\ cm^3}$

Part IV

Answer the 2 questions in th part. Each correct answer will receive 6 credits. Clearly indicate the necessary steps, incl ding appropriate formula substitutions, diagrams, graphs, charts, etc. Utilize the informati n provided for each question to determine your answer. Note that diagrams are not neces rily drawn to scale. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. All answers should be written in pen, except for graphs nd drawings, which should be done in pencil. [12]

35 In quadrilateral $ABCD$, $\overline{AB} \cong \overline{C}$ $\overline{D}$, $\overline{AB} \parallel \overline{CD}$, and $\overline{BF}$ and $\overline{DE}$ are perpendicular to diagonal $\overline{AC}$ at points F and E.



Prove: $\overline{AE} \cong \overline{CF}$

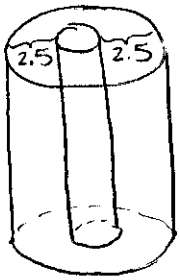
S	R
① Quad ABCD, AB ≅ CD AB ∥ CD BF & DE ⊥ to AC	① Given
② ∠1 ≅ ∠2	② ∥ lines cut by a transversal form ≅ alternate interior ∠s
③ ∠3 & ∠4 are right ∠s	③ ⊥ lines form right ∠s
④ ∠3 ≅ ∠4	④ All right ∠s are ≅
⑤ △DEC ≅ △BFA	⑤ AAS ≅ AAS
⑥ $\overline{AF} \cong \overline{CE}$	⑥ CPCTC
⑦ $\overline{AC} \cong \overline{AC}$	⑦ Reflexive
⑧ $\overline{AC} \cong \overline{AC}$ $-\ \overline{AF} \cong \overline{CE}$ $\overline{CF} \cong \overline{AE}$	⑧ Subtraction Postulate

36 New streetlights will be installed along a section of the highway. The posts for the streetlights will be 7.5 m tall and made of aluminum. The city can choose to buy the posts shaped like cylinders or the posts shaped like rectangular prisms. The cylindrical posts have a hollow core, with aluminum 2.5 cm thick, and an outer diameter of 53.4 cm. The rectangular-prism posts have a hollow core, with aluminum 2.5 cm thick, and a square base that measures 40 cm on each side.

The density of aluminum is 2.7 g/cm^3, and the cost of aluminum is \$0.38 per kilogram.

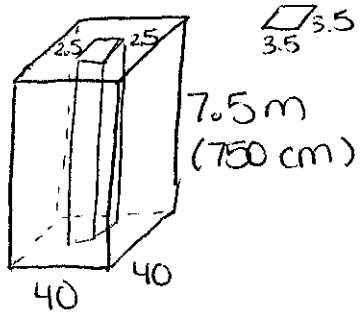
If all posts must be the same shape, which post design will cost the town less?

How much money will be saved per streetlight post with the less expensive design?



D=53.4 D=48.4
r=26.7 r=24.2

$V_{Big} - V_{Little}$
$\pi r^2 h - \pi r^2 h$
$\pi(26.7)^2(750) - \pi(24.2)^2(750)$
299825.7489



$V_{Big} - V_{Little}$
$lwh - lwh$
$(40)(40)(750) - (35)(35)(750)$
281250

$D = \frac{M}{V}$

$\frac{2.7}{1} = \frac{M}{299825.7489}$

$M = \frac{809529.522}{1000}$ g

809.529522 Kg(.38) =
\$307.62

$\frac{2.7}{1} = \frac{M}{281250}$

$M = \frac{759375}{1000}$ g

759.375 Kg(.38) =
\$288.56

307.62
-288.56
\$19.06

Work space for question 36 is continued on the next page.